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**Examination: B.Sc.(H)**

**Subject: Statistics**

**Paper: Time Series Analysis**

**Code: 32377905**

**Ques 1.**

**THEORY (Formula Used)**

METHOD OF PARTIAL SUMS:

Equation for modified exponential curve: yt = a + bct (1)

The given time series data is split up into 3 equal parts, each containing n consecutive values of yt corresponding to t=1,2,…,n ; t=n+1,n+2,…,2n ; t=2n+1,2n+2,…,3n. Let S1, S2 and S3 represent the partial sums of the 3 parts respectively, such that,

* S1 =
* S2 =
* S3 =

Substituting for yt from equation (1), we get the values of a, b and c.

* c =( )1/n
* b =
* a = []

GROMPERTZ CURVE:

The Grompertz curve describes a trend in which the growth increments of the logarithms are declining by a constant percentage. Thus the natural values of the trend would show a declining ratio of increase, but the ratio does not decrease by either a constant amount or a constant percentage.

Equation: yt = a + bc^t

Taking log both sides,

log yt = log a + ct \* log b

Let, log yt = Yt, log a = A, log b = B

Therefore, we get

Yt = A + Bct

The above equation is comparable to the equation of modified exponential curve.

**CALCULATIONS:**

Table 1.1



Table 1.2 Table 1.3



Graph 1.1

**RESULT:**

* Trend values using Grompertz Curve (method of partial sums) have been calculated and shown in Table 1.1.
* The trend values have been plotted along with the given values in Graph 1.1.
* Correlation coefficient between the given amount and the estimated amount is:

R2 = 0.965275084.

**CONCLUSION:**

* The R2 value calculated is almost equal to 1 (0.965275084). This indicates that the values estimated are almost equal to the given values.

**Ques 3.**

**THEORY (Formula Used)**

Correlogram: A correlogram is a visual way to show autocorrelation in the data that changes over time (i.e. time series data).

Correlogram of Moving Average: For a moving average of extent m, with weights

(a1, a2, …am) of random components (εi ; i=1,2,…), the generated series is given by:

Where εi’s are iid N(0, σ2). Thus,

E(yi) = 0 = E(yi+k)

And

Similarly,



**CALCULATIONS:**

Table 3.1



Table 3.2



Graph 3.1

**RESULT:**

* The value of autocorrelation (rk)for the order k have been shown in Table 3.2
* Graph 3.1 shows the correlogram plotted for the moving average.

**Ques 4.**

**THEORY (Formula Used)**

VARIATE DIFFERENCE METHOD:

1. We first have to find the values of Δyt, Δ2yt, Δ3yt … for the given values of t and yt.
2. Then compute: V1 =, V2 =
3. If, let us say, V1 and V2 do not differ significantly than either of them can be regarded as an estimate of V. Otherwise, calculate V3, V4, … till two successive estimates of V are homogeneous.



**CALCULATIONS:**

Table 4.1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| t | yt | Δyt | Δ2yt | Δ3yt | Δ4yt | Δ5yt | Δ6yt |
| 1 | 1006 |  |  |  |  |  |  |
|  |  | 112 |  |  |  |  |  |
| 2 | 1118 |  | -106 |  |  |  |  |
|  |  | 6 |  | -30 |  |  |  |
| 3 | 1124 |  | -136 |  | 184 |  |  |
|  |  | -130 |  | 154 |  | -238 |  |
| 4 | 994 |  | 18 |  | -54 |  | 67 |
|  |  | -112 |  | 100 |  | -171 |  |
| 5 | 882 |  | 118 |  | -225 |  | 530 |
|  |  | 6 |  | -125 |  | 359 |  |
| 6 | 888 |  | -7 |  | 134 |  | -605 |
|  |  | -1 |  | 9 |  | -246 |  |
| 7 | 887 |  | 2 |  | -112 |  | 642 |
|  |  | 1 |  | -103 |  | 396 |  |
| 8 | 888 |  | -101 |  | 284 |  | -791 |
|  |  | -100 |  | 181 |  | -395 |  |
| 9 | 788 |  | 80 |  | -111 |  | 373 |
|  |  | -20 |  | 70 |  | -22 |  |
| 10 | 768 |  | 150 |  | -133 |  | -66 |
|  |  | 130 |  | -63 |  | -88 |  |
| 11 | 898 |  | 87 |  | -221 |  | 739 |
|  |  | 217 |  | -284 |  | 651 |  |
| 12 | 1115 |  | -197 |  | 430 |  | -1253 |
|  |  | 20 |  | 146 |  | -602 |  |
| 13 | 1135 |  | -51 |  | -172 |  | 1089 |
|  |  | -31 |  | -26 |  | 487 |  |
| 14 | 1104 |  | -77 |  | 315 |  | -1437 |
|  |  | -108 |  | 289 |  | -950 |  |
| 15 | 996 |  | 212 |  | -635 |  | 1995 |
|  |  | 104 |  | -346 |  | 1045 |  |
| 16 | 1100 |  | -134 |  | 410 |  | -1561 |
|  |  | -30 |  | 64 |  | -516 |  |
| 17 | 1070 |  | -70 |  | -106 |  | 1140 |
|  |  | -100 |  | -42 |  | 624 |  |
| 18 | 970 |  | -112 |  | 518 |  | -2054 |
|  |  | -212 |  | 476 |  | -1430 |  |
| 19 | 758 |  | 364 |  | -912 |  | 3035 |
|  |  | 152 |  | -436 |  | 1605 |  |
| 20 | 910 |  | -72 |  | 693 |  | -3482 |
|  |  | 80 |  | 257 |  | -1877 |  |
| 21 | 990 |  | 185 |  | -1184 |  | 5315 |
|  |  | 265 |  | -927 |  | 3438 |  |
| 22 | 1255 |  | -742 |  | 2254 |  | -6796 |
|  |  | -477 |  | 1327 |  | -3358 |  |
| 23 | 778 |  | 585 |  | -1104 |  | 2620 |
|  |  | 108 |  | 223 |  | -738 |  |
| 24 | 886 |  | 808 |  | -1842 |  | 5098 |
|  |  | 916 |  | -1619 |  | 4360 |  |
| 25 | 1802 |  | -811 |  | 2518 |  | -7888 |
|  |  | 105 |  | 899 |  | -3528 |  |
| 26 | 1907 |  | 88 |  | -1010 |  | 5202 |
|  |  | 193 |  | -111 |  | 1674 |  |
| 27 | 2100 |  | -23 |  | 664 |  | -4333 |
|  |  | 170 |  | 553 |  | -2659 |  |
| 28 | 2270 |  | 530 |  | -1995 |  | 7372 |
|  |  | 700 |  | -1442 |  | 4713 |  |
| 29 | 2970 |  | -912 |  | 2718 |  | -8943 |
|  |  | -212 |  | 1276 |  | -4230 |  |
| 30 | 2758 |  | 364 |  | -1512 |  | 4635 |
|  |  | 152 |  | -236 |  | 405 |  |
| 31 | 2910 |  | 128 |  | -1107 |  | 3718 |
|  |  | 280 |  | -1343 |  | 4123 |  |
| 32 | 3190 |  | -1215 |  | 3016 |  | -8685 |
|  |  | -935 |  | 1673 |  | -4562 |  |
| 33 | 2255 |  | 458 |  | -1546 |  | 6204 |
|  |  | -477 |  | 127 |  | 1642 |  |
| 34 | 1778 |  | 585 |  | 96 |  | -3109 |
|  |  | 108 |  | 223 |  | -1467 |  |
| 35 | 1886 |  | 808 |  | -1371 |  | 3870 |
|  |  | 916 |  | -1148 |  | 2403 |  |
| 36 | 2802 |  | -340 |  | 1032 |  | -2025 |
|  |  | 576 |  | -116 |  | 378 |  |
| 37 | 3378 |  | -456 |  | 1410 |  |  |
|  |  | 120 |  | 1294 |  |  |  |
| 38 | 3498 |  | 838 |  |  |  |  |
|  |  | 958 |  |  |  |  |  |
| 39 | 4456 |  |  |  |  |  |  |
| SUM= |  | 3450 | 846 | 944 | 1324 | 1226 |  |

Table 4.2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | (Δyt)2 | (Δ2yt)2 | (Δ3yt)2 | (Δ4yt)2 | (Δ5yt)2 | (Δ6yt)2 |
|  |  |  |  |  |  |  |
|  | 12544 |  |  |  |  |  |
|  |  | 11236 |  |  |  |  |
|  | 36 |  | 900 |  |  |  |
|  |  | 18496 |  | 33856 |  |  |
|  | 16900 |  | 23716 |  | 56644 |  |
|  |  | 324 |  | 2916 |  | 4489 |
|  | 12544 |  | 10000 |  | 29241 |  |
|  |  | 13924 |  | 50625 |  | 280900 |
|  | 36 |  | 15625 |  | 128881 |  |
|  |  | 49 |  | 17956 |  | 366025 |
|  | 1 |  | 81 |  | 60516 |  |
|  |  | 4 |  | 12544 |  | 412164 |
|  | 1 |  | 10609 |  | 156816 |  |
|  |  | 10201 |  | 80656 |  | 625681 |
|  | 10000 |  | 32761 |  | 156025 |  |
|  |  | 6400 |  | 12321 |  | 139129 |
|  | 400 |  | 4900 |  | 484 |  |
|  |  | 22500 |  | 17689 |  | 4356 |
|  | 16900 |  | 3969 |  | 7744 |  |
|  |  | 7569 |  | 48841 |  | 546121 |
|  | 47089 |  | 80656 |  | 423801 |  |
|  |  | 38809 |  | 184900 |  | 1570009 |
|  | 400 |  | 21316 |  | 362404 |  |
|  |  | 2601 |  | 29584 |  | 1185921 |
|  | 961 |  | 676 |  | 237169 |  |
|  |  | 5929 |  | 99225 |  | 2064969 |
|  | 11664 |  | 83521 |  | 902500 |  |
|  |  | 44944 |  | 403225 |  | 3980025 |
|  | 10816 |  | 119716 |  | 1092025 |  |
|  |  | 17956 |  | 168100 |  | 2436721 |
|  | 900 |  | 4096 |  | 266256 |  |
|  |  | 4900 |  | 11236 |  | 1299600 |
|  | 10000 |  | 1764 |  | 389376 |  |
|  |  | 12544 |  | 268324 |  | 4218916 |
|  | 44944 |  | 226576 |  | 2044900 |  |
|  |  | 132496 |  | 831744 |  | 9211225 |
|  | 23104 |  | 190096 |  | 2576025 |  |
|  |  | 5184 |  | 480249 |  | 12124324 |
|  | 6400 |  | 66049 |  | 3523129 |  |
|  |  | 34225 |  | 1401856 |  | 28249225 |
|  | 70225 |  | 859329 |  | 11819844 |  |
|  |  | 550564 |  | 5080516 |  | 46185616 |
|  | 227529 |  | 1760929 |  | 11276164 |  |
|  |  | 342225 |  | 1218816 |  | 6864400 |
|  | 11664 |  | 49729 |  | 544644 |  |
|  |  | 652864 |  | 3392964 |  | 25989604 |
|  | 839056 |  | 2621161 |  | 19009600 |  |
|  |  | 657721 |  | 6340324 |  | 62220544 |
|  | 11025 |  | 808201 |  | 12446784 |  |
|  |  | 7744 |  | 1020100 |  | 27060804 |
|  | 37249 |  | 12321 |  | 2802276 |  |
|  |  | 529 |  | 440896 |  | 18774889 |
|  | 28900 |  | 305809 |  | 7070281 |  |
|  |  | 280900 |  | 3980025 |  | 54346384 |
|  | 490000 |  | 2079364 |  | 22212369 |  |
|  |  | 831744 |  | 7387524 |  | 79977249 |
|  | 44944 |  | 1628176 |  | 17892900 |  |
|  |  | 132496 |  | 2286144 |  | 21483225 |
|  | 23104 |  | 55696 |  | 164025 |  |
|  |  | 16384 |  | 1225449 |  | 13823524 |
|  | 78400 |  | 1803649 |  | 16999129 |  |
|  |  | 1476225 |  | 9096256 |  | 75429225 |
|  | 874225 |  | 2798929 |  | 20811844 |  |
|  |  | 209764 |  | 2390116 |  | 38489616 |
|  | 227529 |  | 16129 |  | 2696164 |  |
|  |  | 342225 |  | 9216 |  | 9665881 |
|  | 11664 |  | 49729 |  | 2152089 |  |
|  |  | 652864 |  | 1879641 |  | 14976900 |
|  | 839056 |  | 1317904 |  | 5774409 |  |
|  |  | 115600 |  | 1065024 |  | 4100625 |
|  | 331776 |  | 13456 |  | 142884 |  |
|  |  | 207936 |  | 1988100 |  |  |
|  | 14400 |  | 1674436 |  |  |  |
|  |  | 702244 |  |  |  |  |
|  | 917764 |  |  |  |  |  |
| SUM= | 5304150 | 7570320 | 18751974 | 52956958 | 166229342 | 568108286 |

Table 4.3

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| μ '2(Δyt) | μ '2(Δ2yt) | μ '2(Δ3yt) | μ '2(Δ4yt) | μ '2(Δ5yt) | μ '2(Δ6yt) |
| 139582.895 | 204603.243 | 520888.167 | 1513055.943 | 4889098.294 | 17215402.606 |
|  |  |  |  |  |  |
| Table 4.4 |  |  |  |  |  |
| V1 | V2 | V3 | V4 | V5 | V6 |
| 69791.447 | 34100.541 | 26044.408 | 21615.085 | 19401.184 | 18631.388 |
|  |  |  |  |  |  |
| Table 4.5 |  |  |  |  |  |
| H(1,39) | H(2,39) | H(3,39) | H(4,39) | H(5,39) |  |
| 11.995 | 15.818 | 18.489 | 20.351 | 21.617 |  |
|  |  |  |  |  |  |
| Table 4.6 |  |  |  |  |  |
| R1 | R2 | R3 | R4 | R5 |  |
| 6.134 | 3.737 | 3.144 | 2.084 | 0.858 |  |

**RESULT:**

We have:

|R1| = 6.134, |R2| = 3.737, |R3| = 3.144, |R4| = 2.084, |R5| = 0.858

Now,

* As |R1| > 1.96, therefore difference between V1 and V2 is significant.
* As |R2| > 1.96, therefore difference between V2 and V3 is significant.
* As |R3| > 1.96, therefore difference between V3 and V4 is significant.
* As |R4| > 1.96, therefore difference between V4 and V5 is significant.
* As |R5| < 1.96, therefore difference between V5 and V6 is not significant

**CONCLUSION:**

As the difference between V5 and V6 is insignificant, hence we can say that any of the two can be taken as the variance of the random component.

Therefore,

Variance of the random component = 19401.184

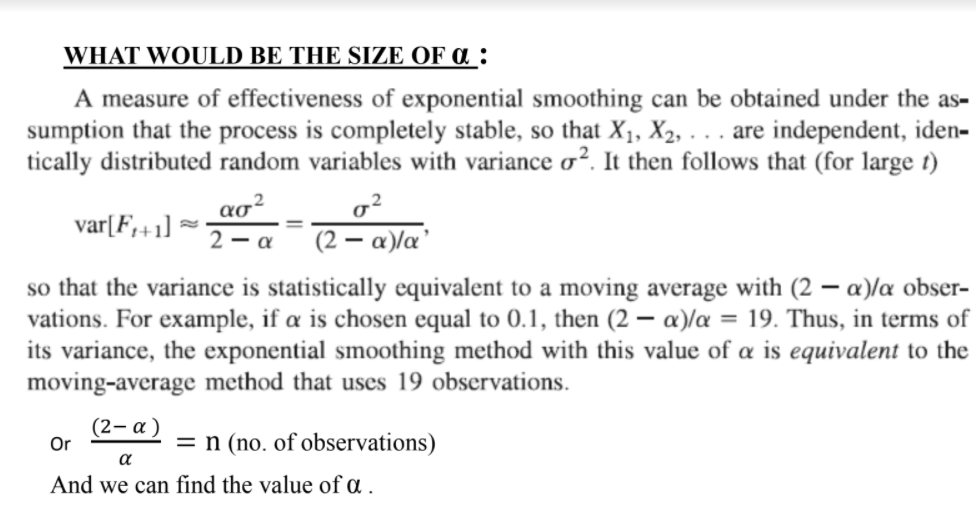
OR

Variance of the random component = 18631.388

**Ques 5.**

**THEORY (Formula Used)**

* Exponential smoothing is a time series forecasting method for univariate data. Forecasts produced using exponential smoothing methods are weighted averages of past observations, with the weights decaying exponentially as the observations get older.
* Equation Ft+1 = αDt + (1- α)Ft, is used to forecast the values for the next months. Here, F is forecasted value, D is demand and α is smoothing constant.
* For t=1, Ft is the first forecasting value, obtained by calculating median of the first 6 observations.



**CALCULATIONS**

Table 5.1

|  |  |
| --- | --- |
| Given Data | |
| t | yt |
| 1990 | 23832 |
| 1991 | 26410 |
| 1992 | 24735 |
| 1993 | 21778 |
| 1994 | 24104 |
| 1995 | 28846 |
| 1996 | 29010 |
| 1997 | 31749 |
| 1998 | 35508 |
| 1999 | 31830 |
| 2000 | 25510 |
| 2001 | 24333 |
| 2002 | 29709 |
| 2003 | 36543 |
| 2004 | 45218 |
| 2005 | 22432 |
| 2006 | 32146 |
| 2007 | 44452 |
| 2008 | 26111 |
| 2009 | 50675 |
| 2010 | 36313 |
| 2011 | 37552 |
| 2012 | 42794 |
| 2013 | 45476 |
| 2014 | 44069 |
| 2015 | 47052 |
| 2016 | 44333 |
| 2017 | 46169 |
| 2018 | 54110 |
| 2019 | 49850 |
| 2020 |  |
| 2021 |  |
| 2022 |  |

Table 5.2

|  |
| --- |
| Exponential Smoothing |
| ɑ= 0.64 |
| 28846 |
| 28522.51613 |
| 28386.22477 |
| 28150.66188 |
| 27739.5224 |
| 27504.97257 |
| 27591.49047 |
| 27683.00721 |
| 27945.32933 |
| 28433.24356 |
| 28652.38914 |
| 28449.65436 |
| 28184.06375 |
| 28282.44674 |
| 28815.38566 |
| 29873.61884 |
| 29393.5144 |
| 29571.09412 |
| 30531.15256 |
| 30245.98143 |
| 31563.98263 |
| 31870.37084 |
| 32236.92756 |
| 32918.02901 |
| 33728.22069 |
| 34395.36774 |
| 35211.92466 |
| 35800.38113 |
| 36469.32429 |
| 37607.4324 |
| 38397.27547 |
| 39136.16092 |
| 39827.37635 |

|  |  |
| --- | --- |
| ɑ= | 0.064516 |
| 1-ɑ= | 0.935484 |

Graph 5.1

**RESULT:**

* Table 5.2 shows the exponential smoothing done for the data given in Table 5.1
* Alpha was calculated to be 0.64
* Graph 5.1 has been plotted to compare the values obtained for different smoothing constants.
* Value forecasted for the next 3 years is:

|  |  |
| --- | --- |
| 2020 | 3216.129 |
| 2021 | 6224.766 |
| 2022 | 9039.297 |

**CONCLUSION:**

* The logical weights are assigned to all the variables, means more weight has been given to recent values and less to previous.
* Hence we can say that the value of α should be minimum and logical to get the best forecast value.